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String junctions in B-field background

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Abstract: It has been recently shown that F-theory on $K3$ with background B-fields (NSNS and RR 2-forms) is dual to the CHL string in 8 dimensions. In this paper, we reexamine this duality in terms of string junctions in type-IIB string theory. It is in particular stressed that certain 7-brane configurations produce Sp gauge groups in a novel way.

Keywords: F-Theory, String Duality, Brane Dynamics in Gauge Theories, Gauge Symmetry.

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Contents

1. Introduction

In recent developments in string theory, the notion of duality has played a crucial role in the understanding of its non-perturbative properties. Some of the dualities are clearly understood in terms of twelve dimensional F-theory [1, 2]. The "definition" of F-theory is as follows. We consider F-theory compactified on a Calabi-Yau $(n+1)$-fold X_{n+1} which admits an elliptic fibration $\pi : X_{n+1} \to B_n$, where B_n is an n complex dimensional manifold. Then, this theory is dual to type-IIB string theory compactified on B_n with 7-branes wrapped on divisors in B_n. Via this type-IIB description, various string vacua can be analyzed in the context of F-theory.

The power of F-theory would be well illustrated by the duality between F-theory and heterotic string. For instance, F-theory compactified on an elliptically fibered $K3$ surface is dual to heterotic string compactified on a 2-torus. In this duality, the Narain moduli space on the heterotic side, which contains the kähler and complex moduli of the 2-torus as well as the Wilson line data, coincides with the complex structure moduli space of the $K3$ on the F-theory side. The origin of gauge symmetry in 8 dimensions is quite different in these dual theories. In heterotic theory, gauge symmetry comes from the breaking of the $E_8 \times E_8$ or SO(32) gauge group to its subgroups. On the other hand, F-theory acquires unbroken gauge symmetry when the $K3$ surface develops the corresponding ADE-type singularity.

The definition of F-theory implies that the 8 dimensional theory given above is represented also by type-IIB string theory on $B_1 = \mathbf{P}^1$ with 24 7-branes located at points on the $\mathbf{P}^1$. The condition that the $K3$ surface in the F-theory description develops a singularity, namely the condition that a gauge symmetry enhancement occurs, corresponds to the situation where in type-IIB theory several 7-branes coalesce at the same position. The perturbative spectra of heterotic string are identified with string junctions stretched between 7-branes. The 7-brane configurations which give rise to ADE-type gauge groups have been obtained in [3, 4, 5].

By modding out $E_8 \times E_8$ heterotic string compactified on a circle by some involution, we can construct another heterotic theory which is now referred to as the CHL string [6, 7]. It has been recently found by Bershadsky, Pantev and Sadov that the CHL string in 8 dimensions also has a dual description in terms of F-theory on a $K3$ [8]. The point is that incorporating B-field background in the F-theory part is equivalent to the CHL involution. Specifically, we switch on NSNS and/or RR 2-form fluxes along $\mathbf{P}^1$ so that the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type-IIB string is broken to its subgroup $\Gamma_0(2)$ [8, 9]. Then, the complex moduli space of the $K3$ is confined to its subspace, which is identical with the moduli space for the CHL string.

It should be noted that Sp gauge groups are realized in the CHL string in 8 dimensions and its F-theory counterpart. This is a remarkable feature absent in the ordinary vacua where no flux is turned on.[1] Our aim in this paper is to analyze string junctions in the presence of non-vanishing B-fields, and to clarify how Sp gauge groups are realized in the type-IIB framework.

The paper is organized as follows. In section 2, we give a brief review of the CHL string in 8 dimensions. In section 3, the F-theory dual description of the CHL string is reviewed. The CHL involution is realized in the F-theory context by the presence of quantized B fluxes. We consider type-IIB string theory in section 4, and translate the geometric data of F-theory into string junctions. The last section is devoted to conclusions and discussions.

2. Review of the CHL string

Here we give a review of the eight-dimensional CHL string theory and its possible gauge symmetry enhancements. In the following argument we take the left-moving sector to be supersymmetric, and we use the bosonic realization for the right-moving sector.

Let us begin with the ordinary toroidal compactification. In the $\mathbb{T}^d$-compactification of the $E_8 \times E_8$ heterotic string we have d periodic coordinates $X^i(z,\bar{z}) = X^i(z) + \tilde{X}^i(\bar{z})$ and sixteen antiholomorphic bosons $X^I(\bar{z})$. Their mode expansions

[1] It has also been found in [10, 11] that Sp gauge groups appear when several D7-branes meet an $\mathrm{O7}^+$-plane.

take the following form:[2]

$$
\begin{aligned}
X^i(z) &= x^i - ip^i \ln z + i\sum_n \frac{\alpha_n^i}{nz^n}\,, \\
\tilde{X}^i(\bar{z}) &= \tilde{x}^i - i\tilde{p}^i \ln \bar{z} + i\sum_n \frac{\tilde{\alpha}_n^i}{n\bar{z}^n}\,, \\
\tilde{X}^I(\bar{z}) &= \tilde{x}^I - i\tilde{p}^I \ln \bar{z} + i\sum_n \frac{\tilde{\alpha}_n^I}{n\bar{z}^n}\,,
\end{aligned} \tag{2.1}
$$

and the canonical quantization leads to the following commutation relations:

$$
\begin{aligned}
\left[x^i, p_j\right] &= \left[\tilde{x}^i, \tilde{p}_j\right] = i\delta^i{}_j\,, \qquad \left[\tilde{x}^I, \tilde{x}^J\right] = i\delta^{IJ}\,, \\
\left[\alpha_m^i, \alpha_n^j\right] &= \left[\tilde{\alpha}_m^i, \tilde{\alpha}_n^j\right] = mg^{ij}\delta_{m+n,0}\,, \qquad \left[\tilde{\alpha}_m^I, \tilde{\alpha}_n^J\right] = m\delta^{IJ}\delta_{m+n,0}\,.
\end{aligned} \tag{2.2}
$$

The momenta $(p_i, \tilde{p}_i, \tilde{p}_I)$ are quantized in the following way:

$$
\begin{aligned}
\tilde{p}_I &= q_I + A_{iI}w^i\,, \\
\tilde{p}_i &= n_i + \left(-g_{ij} - B_{ij} + A_i^I A_j^I\right)\frac{w^j}{2} + A_{iI}q^I\,, \\
p_i &= n_i + \left(g_{ij} - B_{ij} + A_i^I A_j^I\right)\frac{w^j}{2} + A_{iI}q^I\,,
\end{aligned} \tag{2.3}
$$

$$
n_i, w^i \in \mathbb{Z}\,, \qquad \left(q^I\right) \equiv \vec{q} = (\vec{q}_1, \vec{q}_2) \in \Gamma_8 \oplus \Gamma_8\,. \tag{2.4}
$$

Here the (half-)integers (n_i, w^i, q^I) correspond respectively to the momenta, the winding numbers and the gauge charges, and specify a point in the Narain lattice $\Gamma^{d,d+16}$. The background fields g_{ij}, B_{ij}, A_{iI} parameterize the following moduli space of vacua:

$$
O\left(d, d+16, \Gamma^{d,d+16}\right)\backslash O(d, d+16)/(O(d) \times O(d+16))\,.
$$

In fact, if we define the norm of the momentum vector by $p^2 \equiv p_i p^i - \tilde{p}_i\tilde{p}^i - \tilde{p}^I\tilde{p}^I$, it is indeed independent of the moduli:

$$
p^2 \equiv p_i p^i - \tilde{p}_i\tilde{p}^i - \tilde{p}^I\tilde{p}^I = 2n_i w^i - q^I q^I \in 2\mathbb{Z}\,.
$$

A choice of a vacuum uniquely determines the decomposition of $V \equiv \mathbb{R}^{d,d+16}$ into two subspaces $V_\pm$, each having positive and negative-definite norm. The gauge symmetry enhancements occur precisely when some lattice vectors of squared norm -2 are contained in V_-. Thus the analysis of possible gauge enhancements is reduced to a purely mathematical problem.

The CHL string is defined as the asymmetric $\mathbb{Z}_2$ orbifold of the $E_8 \times E_8$ heterotic string theory compactified onto torus. The $\mathbb{Z}_2$ symmetry to be considered is the exchange of two E_8's accompanied by a half-period shift.

[2]Here and throughout the section we use the convention $\alpha' = 2$.

The orbifolding gives rise to two sectors. In the untwisted sector we have the same quantization of the momenta (2.3) and (2.4) as in the usual toroidal compactification, and we only take the $\mathbb{Z}_2$-symmetric states as physical states. In the twisted sector we have

$$(\vec{n}, \vec{w}) \in \left(\mathbb{Z}^d, \mathbb{Z}^d + \vec{v}\right),$$

where $\vec{v}$ denotes the half-period shift vector. Namely if $\vec{v}$ is taken along the first direction we have $w^1 \in \mathbb{Z} + 1/2$. The quantization of the sixteen antiholomorphic bosons gives

$$\vec{q} = (\vec{q}_1, \vec{q}_1) \quad (2\vec{q}_1 \in \Gamma_8)\,, \qquad \tilde{\alpha}^I_m = (-)^{2m} \tilde{\alpha}^{I\pm 8}_m \quad (m \in \mathbb{Z}/2)\,.$$

In order for the orbifolding to be possible, the Wilson line must be invariant under the exchange of two E_8's, namely $A^I_i = A^{I\pm 8}_i$. Hence the moduli space is reduced to the following form:

$$\begin{aligned} & O(d, d+8, \Gamma_{(d)}) \backslash O(d, d+8)/(\mathrm{O}(d) \times O(d+8))\,, \\ & \Gamma_{(d)} \equiv \Gamma_8 \oplus \Gamma^{d-1,d-1}(2) \oplus \Gamma^{1,1}\,, \\ & \Gamma^{d,d}(2) \equiv \left\{\vec{x} = (\xi_i, \eta^i) \in \mathbb{Z}^{2d}\right\}, \quad \vec{x}^2 = 4\xi_i \eta^i\,. \end{aligned} \tag{2.5}$$

Here the lattice $\Gamma_{(d)}$ is the Narain lattice of the CHL string [12], and any automorphism of this lattice is a symmetry of the CHL string theory. The way to associate a point of $\Gamma_{(d)}$ to a state with charges $(n_i, w^i, \vec{q})$ is given as follows:

$$\mathbf{x} \equiv \left(n_i, 2w^i, \vec{Q} \equiv \vec{q}_1 + \vec{q}_2\right) \in \Gamma_{(d)}\,, \qquad \mathbf{x}^2 \equiv 4 n_i w^i - \vec{Q}^2\,.$$

As in the case of ordinary toroidal compactification, a choice of vacuum determines the decomposition of any vector $\mathbf{x}$ into left- and right-moving projection components, $\mathbf{x} = \mathbf{x}_+ + \mathbf{x}_-$.

We define a classification of lattice points $\{\mathbf{x}\}$ as follows:

type 1. $\mathbf{x} \in 2(\Gamma^{1,1} \oplus \Gamma_8) \oplus \Gamma^{d-1,d-1}(2)$.

type 2. $\mathbf{x}$ is not in the above subset, and $\mathbf{x}^2$ is divisible by 4.

type 3. $\mathbf{x}^2$ is not divisible by 4.

In the table below we summarize the classification of string states according to the above definition. Here the half-period shift is chosen in the first direction.

(A)	$2w^1$ even (untwisted sector)	$Q \in 2\Gamma_8$	n_1 even	type 1
(B)			n_1 odd	type 2
(C)		$Q \notin 2\Gamma_8$	$Q^2/2$ even	type 2
(D)			$Q^2/2$ odd	type 3
(E)	$2w^1$ odd (twisted sector)		$n_1 + Q^2/2$ even	type 2
(F)			$n_1 + Q^2/2$ odd	type 3

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A careful evaluation of the mass-shell condition yields the following mass formula: the mass of the lightest state among those corresponding to a given vector $\mathbf{x}$ of type i is given by

$$m^2 = \frac{\mathbf{x}_+^2}{2} = -\frac{\mathbf{x}_-^2}{2} - N_i\,, \quad (N_i = 2, 0, 1 \quad \text{for } i = 1, 2, 3)\,. \tag{2.6}$$

Here N_i are obtained as the sum of the zero-point energy and the term $\sim (\vec{q}_1 - \vec{q}_2)^2$. Remarkably the mass formula depends only on the type of $\mathbf{x}$ and not on any of the additional informations. Hence the massless charged vector bosons arise either from the $\mathbf{x}$ of type 1 and squared norm -4, or of type 3 and squared norm -2.

Let us focus on the $\mathbb{T}^2$ compactification, and take the half-period along the first direction. To study the possible enhancements of gauge symmetry, it is the best way to consider first the charged vector bosons with $\mathbf{x}^2 = -4$, corresponding to long roots. From the previous argument it follows that all the long roots $\mathbf{x}$ arise from the type 1 of the three subsets. Further analysis leads us to the following lemma:

(1) Any two long roots are orthogonal.

(2) The average of any two long roots belongs to $\Gamma_{(2)}$.

These can be proved straightforwardly once we realize that n_2, w^2 must be odd for any long roots. From these lemma it follows that, given the set of long roots $\mathbf{x}_j\ {}_{(j=1,\ldots,2+n)}$, the lattice vectors

$$\{\pm\mathbf{x}_j,\ (\pm\mathbf{x}_i \pm' \mathbf{x}_j)\,/2\}$$

form the root system of $\mathrm{Sp}(2+n)$. Hence the CHL string can realize the Sp type gauge symmetry in 8 dimensions.

The set of long roots for $\mathrm{Sp}(2+n)$ can be explicitly constructed in the following way [7]. Take an A_n sublattice of Γ_8 and choose its basis $e_i\ {}_{(i=1,\cdots n)}$ so that they satisfy $e_i \cdot e_j = 1 + \delta_{ij}$. Then the long roots are expressed as

$$\begin{array}{rl} & \ \ n_1\ \ w^1\ \ n_2\ \ w^2\ \ \vec{Q} \\ \mathbf{x}_1 = & (0,\ 0,\ 1,\ -1,\ 0)\,, \\ \mathbf{x}_2 = & (2,\ -1,\ 1,\ 1,\ 0)\,, \\ \mathbf{x}_{2+j} = & (0,\ -1,\ 1,\ 1,\ 2e_j)\,. \end{array} \tag{2.7}$$

Generically the enhanced gauge symmetry is given by the product of the Sp type level one algebra and the ADE-type level two algebra. For example, using the formula $A_n \times E_{8-n} \subset E_8$ we find that the following enhancements are possible:

$$\mathrm{Sp}(2+n) \times E_{8-n} \quad (n = 0, \ldots, 8)\,, \tag{2.8}$$

where we have used the notation $E_5 = \mathrm{SO}(10)$, $E_4 = \mathrm{SU}(5)$, $E_3 = \mathrm{SU}(3) \times \mathrm{SU}(2)$, $E_2 = \mathrm{SU}(2) \times \mathrm{U}(1)$ and $E_1 = \mathrm{SU}(2)$.

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3. F-theory dual of the CHL string

In this section we present the F-theory compactification to 8 dimensions which precisely yields a dual description of the CHL string explained in the previous section, following [8, 9]. Since the ordinary Narain moduli space is restricted to its subspace due to the CHL involution, we have to find a similar mechanism which freezes some of the complex moduli of $K3$ so as to obtain the same moduli space. This is systematically done by turning on 2-form background fields B taking values in $H^2(\mathbf{P}^1, \mathbb{Z}/2)$.

Let us consider the case where 2-forms $B_{\rm NS}$ and B_R take the values

$$\begin{pmatrix} B_{\rm NS} \\ B_R \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}. \tag{3.1}$$

If $(B_{\rm NS}, B_R)$ is non vanishing, the $\mathrm{SL}(2,\mathbb{Z})$ monodromy group around 7-branes on $\mathbf{P}^1$ must be reduced to a smaller group which keeps $(B_{\rm NS}, B_R)$ invariant modulo $H^2(\mathbf{P}^1,\mathbb{Z})$. In the present case (3.1), the reduced monodromy group is $\Gamma_0(2)$, which is represented by the matrices of the form

$$\Gamma_0(2) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix};\ a,b,d \in \mathbb{Z},\ c \in 2\mathbb{Z},\ ad - bc = 1 \right\}. \tag{3.2}$$

The elliptic $K3$ surface with a section can be expressed by the familiar Weierstrass equation, which in general has the entire monodromy group $\mathrm{SL}(2,\mathbb{Z})$, not $\Gamma_0(2)$. In order to get the elliptic $K3$ with $\Gamma_0(2)$ monodromy, which we denote by X, the Weierstrass equation must take a special form. To find the form of the equation, note that the elliptic fibration of the $K3$ must have one more section which intersects the fiber at the point corresponding to a half-period of the fiber torus. We thus conclude that X is of the form

$$X : y^2 = (x - a_4(z))\left(x^2 + a_4(z)x + b_8(z)\right), \tag{3.3}$$

where z is the coordinate on the $\mathbf{P}^1$ base, and $a_4(z), b_8(z)$ are polynomials of degree 4 and 8, respectively. The Weierstrass form (3.3) indeed has a section $s_1 : (x = a_4(z),\ y = 0)$, in addition to the ordinary one $s_0 : (x = \infty,\ y = \infty)$. If we denote by α and β the two independent 1-cycles of the fiber torus which correspond to NSNS and RR directions, then the additional section s_1 corresponds to the half-period $(1/2)\alpha + 0 \cdot \beta$, in agreement with the quantized flux (3.1).

The discriminant Δ of X is given by

$$\Delta = \left(4b_8 - a_4^2\right)\left(b_8 + 2a_4^2\right)^2. \tag{3.4}$$

This expression shows that X always has eight A_1 type singularities, and eight I_1 type singular fibers. All the eight A_1 singularities are situated at the section s_1.

The $K3$ surface X given by (3.3) with $\Gamma_0(2)$ monodromy can be constructed from another $K3$ surface Y, through the following two steps. First, we mod out Y by an involution σ, to make the "intermediate" $K3$ $\tilde{X} = Y/\sigma$. The involution σ is none other than a geometric realization of the CHL involution. Second, we apply a birational transformation to $\tilde{X}$, yielding the desired $K3$ surface X. Let us start with the construction of Y. We take the $K3$ surface Y to be a double cover of the Hirzebruch surface $\mathbf{F}_0 = \mathbf{P}^1 \times \mathbf{P}^1$, which branches along the zero locus of a section of the line bundle $\mathcal{O}_{\mathbf{F}_0}(4,4)$ over $\mathbf{F}_0$. Denoting the inhomogeneous coordinates on the two $\mathbf{P}^1$ factors in $\mathbf{F}_0$ by s and t, we can represent Y as the hypersurface equation

$$Y : y^2 = \sum_{i,j=0}^{4} a_{i,j} s^i t^j \,. \tag{3.5}$$

Here, $a_{i,j}$ parametrize the complex structure moduli of Y. By fixing either one of s and t, one can see that Y admits two elliptic fibrations $p : Y \to \mathbf{P}^1$ and $q : Y \to \mathbf{P}^1$ whose bases are the two $\mathbf{P}^1$ factors in $\mathbf{F}_0$ parametrized by s and t, respectively.

Let us turn to the first step, the application of the involution σ to Y. The involution σ is defined by the $\mathbb{Z}_2$ action on Y,

$$\sigma : (y, s, t) \longrightarrow (-y, s, -t) \,. \tag{3.6}$$

This means that σ inverses the orientation of one of the $\mathbf{P}^1$ and the elliptic fiber over it simultaneously. Since Y should be invariant under the $\mathbb{Z}_2$ action (3.6) in order to be modded out by σ, Y has to take the form

$$y^2 = \sum_{i=0}^{4} \sum_{j=0}^{2} a_{i,2j} s^i t^{2j} \,. \tag{3.7}$$

The hypersurface (3.7) is embedded in $\mathbb{C}^3$ parametrized by (y, s, t) which is reduced to $\mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$ by the involution σ. The orbifold $\mathbb{C}^2/\mathbb{Z}_2$ is described by the three well-defined coordinates $(u = y^2, v = t^2, w = yt)$ and the relation among them

$$uv = w^2 \,. \tag{3.8}$$

By combining (3.7) with (3.8), we can express $\tilde{X} = Y/\sigma$ by the hypersurface equation parametrized by (w, s, v),

$$\tilde{X} : w^2 = vP(s,v) \,, \quad P(s,v) = \sum_{i=0}^{4} \sum_{j=0}^{2} a_{i,2j} s^i v^j \,. \tag{3.9}$$

Note that, similarly to Y, $\tilde{X}$ can also be interpreted as a double cover of $\mathbf{F}_0$, which is now coordinatized by s and v. The only difference between Y and $\tilde{X}$ is that the branch divisor defining $\tilde{X}$ consists of three components $\tilde{C}_1 : \{v = \infty\}$, $\tilde{C}_2 : \{v = 0\}$ and $\tilde{C}_3 : \{P(s,v) = 0\}$, whereas the branch divisor for a generic Y consists of a single component. In other words, the involution σ has the effect of restricting the complex moduli space of Y, so that the branch divisor defining Y splits to $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$, which are identified with the zero loci of sections of line bundles $q^*\mathcal{O}_{\mathbf{P}^1}(1)$, $q^*\mathcal{O}_{\mathbf{P}^1}(1)$ and $\mathcal{O}_{\mathbf{F}_0}(4,2)$.

The second step is to convert $\tilde{X}$ into X by a birational transformation. To do this, we focus on the fact that $\tilde{X}$ has eight A_1 type singularities, four of which are on $\tilde{C}_1$, and the other four on $\tilde{C}_2$. The four A_1 singularities on $\tilde{C}_1$ comes from the four points where $\tilde{C}_3$ intersects $\tilde{C}_1$. The same is true for the singularities on $\tilde{C}_2$. By blowing up $\mathbf{F}_0$ at the four points on $\tilde{C}_1$, we obtain a surface birational to $\mathbf{F}_0$, whose generic $\mathbf{P}^1$ fiber becomes reducible at the four points on the $\mathbf{P}^1$ base where $\tilde{C}_3$ meets $\tilde{C}_1$. Each of the four reducible fibers consists of two $\mathbf{P}^1$'s, one of which is generated by the blowing up procedure. Then, we blow down the remaining four $\mathbf{P}^1$'s in the reducible fibers. The resulting surface is $\mathbf{F}_4$, on which there exist three divisors C_i $(i = 1, 2, 3)$ which inherit from $\tilde{C}_i$. From the above birational transformation, it is easy to see that C_2 intersects C_3 at eight points, while C_1 does not intersect C_3 at all. Finally, we identify X with the double cover of $\mathbf{F}_4$, whose branch divisor consists of C_i $(i = 1, 2, 3)$. The two sections s_0 and s_1 of X originate from C_1 and C_2, and X indeed has eight A_1 singularities on s_1 which appear from the eight points where C_2 and C_3 intersect. It is also important to note that the elliptic fibration π for X traces back to the elliptic fibration p for Y.

The $K3$ surface X constructed in this way can be represented by eq. (3.3), which has monodromy group $\Gamma_0(2)$. Then we switch on the background flux (3.1), to ensure that the monodromy group of X is not enlarged to $\mathrm{SL}(2,\mathbb{Z})$.

To determine the unbroken gauge symmetry in F-theory on X, we need to examine the relation between the singularity structures of Y and X. In what follows, we shall not make an exhaustive analysis, but illustrate the main idea by just one example. The case to be considered in this paper is an A_{2n-1} type singularity in Y, which is located on a fixed elliptic fiber $\{t = 0\}$ with respect to the fibration q. By appropriately adjusting the coefficients $a_{i,2j}$ in (3.7), we can let Y develop an A_{2n-1} $(n = 1, \ldots, 8)$ singularity at the point $\{s = t = 0\}$. Explicitly, the $K3$ surface Y with such a singularity is given by

$$\begin{aligned}
A_1 &: y^2 = t^4 + t^2 + s^2\,, \\
A_3 &: y^2 = t^4 + st^2 + s^2\,, \\
A_5 &: y^2 = st^4 + t^4 + 2st^2 + s^2\,, \\
A_7 &: y^2 = s^2t^4 + t^4 + 2st^2 + s^2\,, \\
A_9 &: y^2 = s^3t^4 + t^4 + 2st^2 + s^2\,, \\
A_{11} &: y^2 = s^4t^4 + t^4 + 2st^2 + s^2\,, \\
A_{13} &: y^2 = s^4t^4 + s^3t^4 + 2s^2t^4 + t^4 + s^4t^2 + 2s^3t^2 + 2st^2 + s^2\,, \\
A_{15} &: y^2 = s^3t^4 + t^4 + s^4t^2 + 2st^2 + s^2\,.
\end{aligned} \tag{3.10}$$

If we resolve the above A_{2n-1} singularity, there appear, from the singularity, $2n-1$ $\mathbf{P}^1$'s which intersect one another according to the A_{2n-1} Dynkin diagram. Let us denote these $2n-1$ components by S_i ($i=1,\dots,2n-1$). Under the involution σ, the middle component S_n remains unchanged, whereas the other $2n-2$ components are exchanged in the manner

$$S_i \longleftrightarrow S_{2n-i} \quad (i=1,\dots,n-1)\,. \tag{3.11}$$

This suggests that σ acts on S_i's as an outer automorphism of the A_{2n-1} algebra, giving rise to the new algebra C_n.

Therefore we expect that $\mathrm{Sp}(n)$ gauge symmetry enhancement takes place in F-theory on X, when Y has an A_{2n-1} type singularity on $\{t=0\}$ (or on $\{t=\infty\}$). The next problem is then to determine what type of singularity X should develop in order for $\mathrm{Sp}(n)$ gauge symmetry to be produced. This is easily done by just translating the singularity data (3.10) for Y into the one for X. It turns out that $\tilde{X}$ develops an A_3 (resp. D_{n+2} ($n=2,\dots,8$)) singularity when Y has a singularity of the type A_1 (resp. A_{2n-1}). The explicit form of $\tilde{X}$ can be determined by using (3.7), (3.9) and (3.10) to be

$$\begin{aligned}
A_3 &: w^2 = v(v^2+v+s^2)\,,\\
D_4 &: w^2 = v(v^2+sv+s^2)\,,\\
D_5 &: w^2 = v(sv^2+v^2+2sv+s^2)\,,\\
D_6 &: w^2 = v(s^2v^2+v^2+2sv+s^2)\,,\\
D_7 &: w^2 = v(s^3v^2+v^2+2sv+s^2)\,,\\
D_8 &: w^2 = v(s^4v^2+v^2+2sv+s^2)\,,\\
D_9 &: w^2 = v(s^4v^2+s^3v^2+2s^2v^2+v^2+s^4v+2s^3v+2sv+s^2)\,,\\
D_{10} &: w^2 = v(s^3v^2+v^2+s^4v+2sv+s^2)\,.
\end{aligned} \tag{3.12}$$

In the present situation, the singularity structures of X and $\tilde{X}$ are the same. We are thus led to the following dictionary relating the type of singularity in X to gauge symmetry:

singularity in X	gauge symmetry
A_3	$\mathrm{Sp}(1)$
D_{n+2} ($n=2,\dots,8$)	$\mathrm{Sp}(n)$

(3.13)

One might wonder why A_3 (resp. D_{n+2}) singularity does not give rise to SU(4) (resp. SO($2n$+4)), but to Sp gauge symmetry. This is due to the presence of quantized fluxes along the base of X. Recall that X necessarily has eight A_1 singularities and eight I_1 fibers. The A_1 singularities cannot be deformed, since deforming them would force X to have the full monodromy group $\mathrm{SL}(2,\mathbb{Z})$, and therefore conflict with the presence of background fluxes. The A_3 singularity in X is made from the collision of two A_1 singularities. Similarly, the D_{n+2} singularity in X comes from the collision of two A_1 singularities and n I_1 fibers. If the A_1 singularities could be deformed, then

SU(4) or SO$(2n+4)$ gauge symmetry would be generated as it should be. However, since deforming the A_1 singularities is in fact forbidden, it is possible that Sp gauge groups are realized though it seems strange at first sight. In the next section, we will find further evidence that Sp(n) $(n = 1, \ldots, 8)$ gauge symmetry is actually generated, by explicitly constructing the string junctions which realize the simple roots for Sp(n) Lie algebra.

4. String junction

In this section we consider the type-IIB string theory compactified on $B_1 = \mathbf{P}^1$. There are 24 7-branes located at points on the $\mathbf{P}^1$, and we turn on the background B-field as explained in the previous section. The non-zero B-fields do not break the supersymmetry, because the supersymmetry transformation law of fermions contain only their field strengths $H = dB$, which simply vanish in this case. We will focus on the vicinity of the 7-branes realizing the Sp type gauge symmetry. Indeed, such an enhancement of the gauge symmetry is possible in the CHL string theory, and we may as well expect the same symmetry also in the F-theory by constructing the $K3$ surface X from the other $K3$ surface Y by an orbifolding. However, it is not clear whether we can realize Sp(n) gauge symmetry by the above construction of X, since it is only X and not Y which is of physical significance. This is true even when we consider the M-theory and membranes by further compactification on S^1. In the following we explain that Sp(n) $(n \geq 2)$ gauge symmetry can be realized by an SO$(2n+4)$-type fiber singularity, by using string junctions and taking account of the background B-field appropriately. The case of Sp$(1) \simeq$ SU(2) will be discussed at the end of this section.

We follow the notation of [15] and in particular denote by $X_{[p,q]}$ the $[p,q]$ 7-brane or the 7-brane on which $[p,q]$-string can end. We also denote the 7-branes $X_{[1,0]}$, $X_{[1,-1]}$, $X_{[1,1]}$ and $X_{[0,1]}$ as $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$, respectively, for notational simplicity. The monodromy of a $[p,q]$ 7-brane is given by the formula

$$K_{[p,q]} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}, \tag{4.1}$$

and the 7-branes $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ have the following monodromy matrices:

$$\begin{aligned} K_{\mathbf{A}} &= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \qquad K_{\mathbf{B}} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}, \\ K_{\mathbf{C}} &= \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, \qquad K_{\mathbf{D}} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}. \end{aligned} \tag{4.2}$$

The D_{n+2} $(n \geq 2)$ singularity of X (3.12), which is necessary for realizing the Sp(n) gauge symmetry, corresponds to the collection of 7-branes $\mathbf{A}^{n+2}\mathbf{B}\mathbf{C}$ in the type-IIB picture. The D_{n+2} singularity can be deformed to two A_1 singularities and

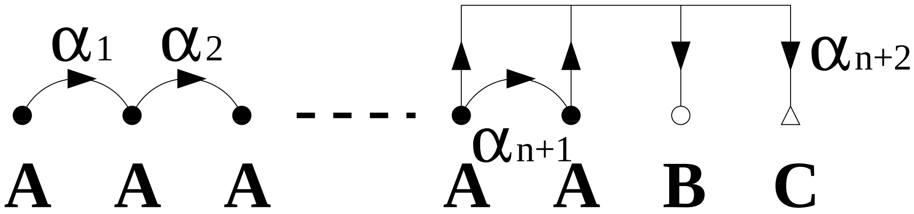

Figure 1: The 7-brane configuration and junctions which realize SO$(2n+4)$.

n I_1 fibers. However, further resolution is not allowed due to the presence of the background B-field. In order to see this in terms of 7-brane configurations, we notice the equivalence between $\mathbf{A}^{n+2}\mathbf{BC}$ and $\mathbf{A}^n\mathbf{D}^2\mathbf{C}^2$. This can be seen as follows. First, by moving a 7-brane across the branch cut of another 7-brane we obtain the following formulae [13]:

$$X_{[p,q]}X_{[r,s]} = X_{[r,s]+(ps-qr)[p,q]}X_{[p,q]} = X_{[r,s]}X_{[p,q]+(ps-qr)[r,s]}\,, \tag{4.3}$$

which give, in particular, $\mathbf{AB} = \mathbf{DA}, \mathbf{AD} = \mathbf{DC}$ and $\mathbf{CA} = \mathbf{DC}$. Using them we obtain

$$\mathbf{A}^{n+2}\mathbf{BC} = \mathbf{A}^{n+1}\mathbf{DAC} = \mathbf{A}^n\mathbf{DCAC} = \mathbf{A}^n\mathbf{D}^2\mathbf{C}^2\,. \tag{4.4}$$

From the new expression for the collection of 7-branes we identify the two A_1 singularities with $\mathbf{D}^2$ and $\mathbf{C}^2$, and I_1 fibers with $\mathbf{A}$'s. Indeed, $K_\mathbf{A}, {K_\mathbf{D}}^2$ and ${K_\mathbf{C}}^2$ are elements of $\Gamma_0(2)$ defined in (3.2) although $K_\mathbf{D}$ and $K_\mathbf{C}$ are not. The VEV of the $n+2$ complex scalars, which are in the multiplets containing the gauge fields for the Cartan subalgebra of the gauge symmetry, represent the positions of the n $\mathbf{A}$'s and relative positions of the two $\mathbf{D}$'s and two $\mathbf{C}$'s on the $\mathbf{P}^1$. Each of these scalars originates from the open string whose two endpoints are on a single 7-brane.

The massless BPS states are represented by string junctions which connect the 7-branes collapsing to a single point on $\mathbf{P}^1$. If there is no background B-field, the string junctions connecting some of the 7-branes $\mathbf{A}^{n+2}\mathbf{BC}$ form the root lattice of SO$(2n+4)$ [3]. We show in figure 1 the junctions which correspond to the simple roots[3] α_i for SO$(2n+4)$.

Now we analyze how the junctions are transformed by the chain of rearrangements of 7-branes that brings $\mathbf{A}^{n+2}\mathbf{BC}$ into $\mathbf{A}^n\mathbf{D}^2\mathbf{C}^2$. By the Hanany-Witten effect [14, 3] a $[p,q]$-string and a $[r,s]$ 7-brane passing through each other create $(ps-qr)$ $[r,s]$-strings between the two. Hence the string junction which have n_1 endpoints on $X_{[p,q]}$ and n_2 on $X_{[r,s]}$ are transformed according to the following identity:

$$n_1[p,q] + n_2[r,s] = n_2\left\{[r,s] + (ps-qr)[p,q]\right\} + \left\{n_1 - (ps-qr)n_2\right\}[p,q]$$

in the process of the rearrangement $X_{[p,q]}X_{[r,s]} = X_{[r,s]+(ps-qr)[p,q]}X_{[p,q]}$. Using this formula we find that the junction corresponding to the α_i in figure 1 is transformed to that in figure 2.

[3]In this paper we use same symbol for a simple root and a junction corresponding to it.

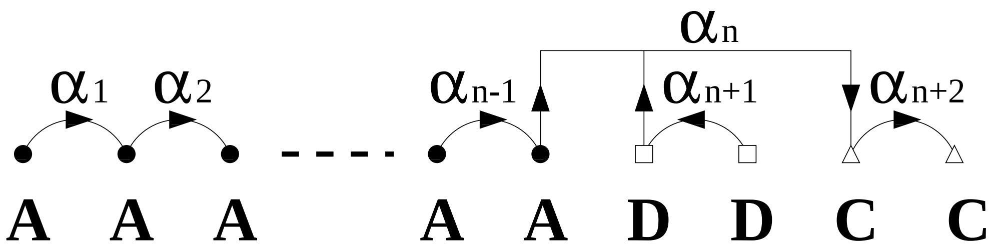

Figure 2: Another representation of the 7-brane configuration and junctions for $\mathrm{SO}(2n+4)$.

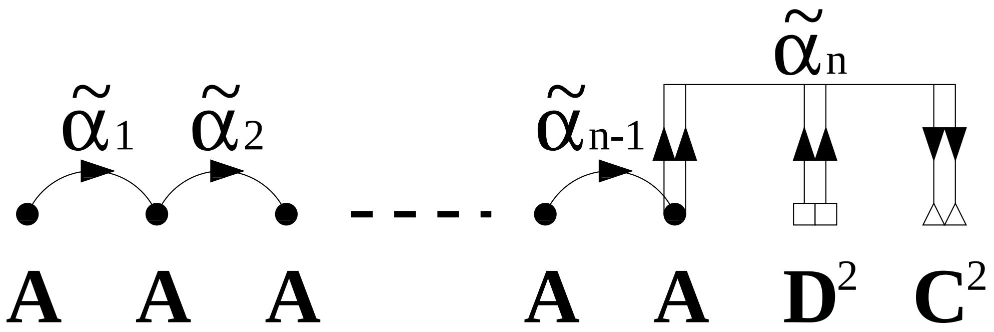

Figure 3: The simple roots for $\mathrm{Sp}(n)$.

Once we turn on the B-field, the gauge fields corresponding to the junction α_j $(n \leq j \leq n+2)$ disappear from the massless spectrum. This is because the subgroup SU(2) of the $\mathrm{SO}(2n+4)$ which is generated by α_j and $-\alpha_j$ contains the fluctuation modes that separate the two **D**'s or two **C**'s. In other words, since the A_1 singularities cannot be deformed, the scalars corresponding to their deformation parameters cannot have non-zero VEV and must not remain massless. On the other hand, junctions which have the same number of endpoints on each of the two **D**'s (and each of the two **C**'s) remain massless, since they correspond to the fluctuations that move $\mathbf{D}^2$ (and $\mathbf{C}^2$) altogether without breaking them into constituents. By taking these rules into account, we conclude that the massless spectrum representing the simple roots for $\mathrm{Sp}(n)$ is generated by $\tilde{\alpha}_i \equiv \alpha_i$ $(1 \leq i \leq n-1)$ and

$$\tilde{\alpha}_n \equiv \alpha_n + (\alpha_n + \alpha_{n+1} + \alpha_{n+2}) , \tag{4.5}$$

as depicted in figure 3.

In particular we can check that $\tilde{\alpha}_{n-1} \cdot \tilde{\alpha}_n = -2$ and that the root $\tilde{\alpha}_n$ has the norm $(\tilde{\alpha}_n)^2 = 4$, namely it is a long root. Note that the inner product of the roots is given by minus the intersection number of the corresponding string junctions, i.e. $\mathbf{J}_{\alpha_i} \cdot \mathbf{J}_{\alpha_j} = -\alpha_i \cdot \alpha_j$ where $\mathbf{J}_{\alpha_i}$ is the junction corresponding to the root α_i. It is also known that, by compactifying further onto S^1 and going to the M-theory picture, the intersection number of junctions is identified with that of membranes wrapping on the corresponding two-cycles in $K3$. Furthermore we expect the BPS condition to be unchanged when we turn on the background B-field [17].[4] Thus we can realize Sp(n) gauge group in type-IIB string theory by 7-branes and the non-vanishing background B-field.

Here we notice that, according to the arguments in [15, 16], the BPS junction $\mathbf{J}$ have to satisfy

$$\mathbf{J} \cdot \mathbf{J} = 2g - 2 + b \geq -2\,, \tag{4.6}$$

where g and b are, the genus and the number of the boundary, of the corresponding two-cycles in $K3$. Thus it seems impossible to have BPS junctions with self-intersection number -4 corresponding to the long roots for Sp(n). However, this is not true because every junction corresponding to the long roots consists of two junctions orthogonal to each other. For example, $\tilde{\alpha}_n$ is expressed as the sum of α_n and $(\alpha_n + \alpha_{n+1} + \alpha_{n+2})$, which would satisfy the BPS condition if the background B-field were turned off. Moreover the two junctions are equivalent as cycles of the corresponding $K3$, since under the presence of the B-field the A_1 singularities cannot be deformed. Thus every junction corresponding to a long root is a bound state of two junctions which cannot be separated due to the background B-field. Note that when we turn off the B-field, it becomes a marginal bound state and can be separated into two BPS junctions.

The 7-brane configuration for Sp(1) has an expression slightly different from the ones for the sequence Sp(n) ($n \geq 2$). The configuration is given by two pairs of $\mathbf{D}$ branes, namely $\mathbf{D}^2\mathbf{D}^2$. As pointed out before, this configuration would give SU(4) gauge symmetry if both pairs of $\mathbf{D}$ branes could be separated. However, we have Sp(1) symmetry here, since the rule concerning massless spectrum allows only one string junction α consisting of two coincident D-strings, which emanate from one of the two $\mathbf{D}^2$'s and get absorbed into the other as depicted in figure 4. We identify this junction with the simple root for Sp(1).

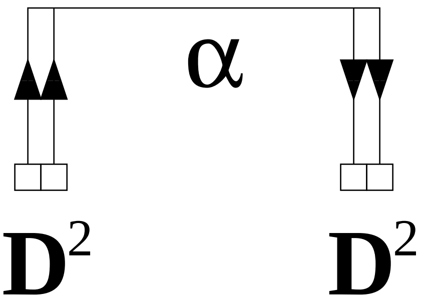

Figure 4: The 7-brane configuration and simple root for Sp(1).

The O7$^+$-plane has also been used to produce Sp(n) gauge group, and a certain dual description of CHL string theory contains it [9, 10]. In this description, a modification of the BPS condition of string junction is required [11]. In fact this is same as our result $\mathbf{JJ} = -4$ for the long root. The O7$^+$-plane might be realized in our picture by eliminating the B-field by a singular gauge transformation and localize it to a single point on $\mathbf{P}^1$. By identifying the O7$^+$-plane with the point onto which the non-zero B-field is localized, we might be able to prove the equivalence between the F-theory with quantized flux and the description with the O7$^+$-plane.

[4] The junctions corresponding to the frozen moduli are also expected to be BPS, but they do not yield massless states according to the argument in the previous paragraph.

5. Conclusions and discussions

In this paper we have clarified how the Sp type gauge symmetry can be realized in type-IIB string theory on $\mathbf{P}^1$ with quantized B flux. In the presence of the B flux the monodromy around the 7-branes are restricted to the subgroup $\Gamma_0(2)$ of $\mathrm{SL}(2,\mathbb{Z})$, and due to this condition some 7-branes are forced to be bound together. The analysis of the string junctions on the collapsed 7-branes shows that, by excluding the junctions corresponding to the deformation that would enlarge the monodromy group, we can obtain $\mathrm{Sp}(n)$ gauge group out of the D_{n+2} singular fiber. Some junctions in the root system of $\mathrm{Sp}(n)$ have the self-intersection -4 and at first sight they cannot be BPS saturated. However, a careful look into the corresponding cycles in $K3$ leads us to realize that the long roots always correspond to two degenerate cycles, so that they are BPS saturated.

It would be a very interesting problem to see the full correspondence between the CHL string and type-IIB string theory with non-zero B-field. To see this, note that the gauge groups of the type (2.8) is possible in the CHL string. However the singular fiber of the type E_6 or E_8 is impossible in the type-IIB string with the B-field, since the monodromies around the corresponding collections of 7-branes are not in $\Gamma_0(2)$. In [8] the realization of such gauge groups as well as $\mathrm{Sp}(9), \mathrm{Sp}(10)$ has been given in a very complicated way. Some enhancements require that the monodromy group become further smaller to the group $\Gamma(2)$. It would be interesting to study how such gauge groups are realized by 7-branes and string junctions in the type-IIB string theory.

On the other hand, according to our construction of Sp gauge symmetry the gauge groups of the form $\mathrm{Sp} \times \mathrm{Sp}$ are apparently possible, which is in contradiction with the result in the CHL string and the description with the $\mathrm{O7}^+$-plane. In the CHL string we can easily see that one cannot realize the product of Sp's, since by the lemma (2) of section 2 the embedding of the root lattice of $\mathrm{Sp} \times \mathrm{Sp}$ in $\Gamma_{(2)}$ would result in the embedding of the larger Sp gauge group. Translating this fact into our framework we must conclude that, if we realize two Sp singularity at two different points on $\mathbf{P}^1$ we obtain a large Sp group, with some junctions linking two distant fiber singularities becoming massless. At present we have no idea for resolving this discrepancy.

Acknowledgments

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